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Spatial Stability of Density Stratified Shear Flows through a Porous Medium

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ABSTRACT:
In this paper, the spatial stability of shear flow in a porous medium has been discussed. Fluid has been considered incompressible and the porous medium isotropic and homogenous everywhere. It is assumed that the medium obeys the Boussinesq approximation.
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INTRODUCTION:
Flow of density stratified fluid through a porous medium and the stability problem there of have been of significant importance in literature. Contributions to the problem of stability in a porous medium are well summarized in the books by **Scheidegger** (8) and **Yih** (10).

Undoubtedly the stability of the fluid interfaces moving in a porous medium is of significant importance for the ground water petroleum production engineering, civil engineering etc. Extensive studies have been conducted on the stability of the interface between two fluids of different densities and viscosities through porous media when there is movement or displacement perpendicular to the interface as well as parallel to the interface. Several authors have endeavored to study the thermal instability of a fluid saturated porous layer. An excellent review of the literature is provided by **Joseph** (7)**.** In all these analysis, the fluid flow has been assumed to be governed by **Darcy's** law. A general argument has been advanced since the experimental findings of **Darcy** (1) that the inclusion of inertia is not interesting for the physics of flow through porous medium. But there are situations in which a departure from **Darcy's** law and the inertia effects not included in **Darcy's** model may become sufficient.

Jaimala and Agarwal (5) investigated the stability of a density stratified fluid with horizontal through a porous medium .They obtained semi-circle type bounds on the complex wave velocity of unstable modes (if exist) under conditions. They discussed the temporal stability of the system. In this chapter the same problem has been taken for the discussion in view of spatial stability. **Goel, Agrawal and Jaimala** (2) discussed the shear flow instability of an incompressible visco-elastic second order fluid in a porous medium. **Sunil, Sharma and Chandel** (9) discussed the stability of stratified Rivlin-Ericksen viscoelastic fluid permeated with suspended particles and a variable magnetic field in porous medium. **Jaimala, Nidhi and Agarwal** (6) examined the hydrodynamic instability of micropolar fluids in porous medium both analytically and numerically. They showed that the micropolar parameter has stabilizing as well as destabilizing effect, J_0 has stabilizing character while the permeability has destabilizing character.

Dua, Goel and Hari Kishan (3) discussed the spatial stability of hydromagnetic swirling flows**. Hari Kishan, Nirmal Kumari and Naresh Kumar (4)** discussed the spatial stability of homogeneous shear flows in sea straits.

In this paper, the spatial stability of density stratified shear flows through a porous medium has been discussed. Fluid has been considered incompressible and the porous medium isotropic and homogenous everywhere following the Boussinesq approximation law.

FORMULATION OF THE PROBLEM:

A fluid saturated porous medium bounded by two infinite parallel plates situated at a distance apart is considered. The x-axis is taken along the main flow in the lower plane and the y-axis is taken perpendicular to the planes. The solid incompressible substrate has a constant porosity ϕ and a constant permeability K. A consideration of theoretical possibilities of the structure of porous media one realize that a general correlation between porosity and permeability do not exist. Porosity is the ratio of the volume of unit pore to that of the unit cell and it will be independent of the radius R of the uniform spheres comprising the assemblage cannot alone provide an accurate indication of its permeability. It is possible for two porous media of the same porosity to have different permeability's. The following simplifying assumptions have been taken in the present analysis:

(i) The saturated fluid is incompressible and all the physical properties of the fluid except the density are constant.

(ii) The porous medium is everywhere isotropic and homogenous.

(iii) The medium obeys the Boussinesq approximation which states that the variations of density in the equations of motion can safely be ignored everywhere in its association with the external force.

The theory of porous layer is based mainly on Darey's law which is applicable to the steady state flows when inertial effects are negligible. Since we wish to study a flow in which inertial effects are included and substantial derivative of $\bar{u}$ is not zero, we have assumed that under such conditions the drag on the fluid can still be approximated by Darey's Law. Thus the physical system under consideration obeys the following fundamental equations:

$$\Delta.\bar{u} = 0, \qquad \ldots(1)$$

$$\frac{\rho}{\phi}\frac{\partial \bar{u}}{\partial t} + \frac{1}{\phi^2}\rho\bar{u}.\Delta\bar{u} = -\Delta\rho - \frac{\nu\rho\bar{u}}{k} - g\rho\bar{\lambda}, \qquad \ldots(2)$$

and $$\frac{\partial \rho}{\partial t} + \frac{1}{\phi}\bar{u}.\nabla\rho = 0, \qquad \ldots(3)$$

where g is the acceleration due to gravity $\bar{\lambda} = (0,1,0)$ is the unit vector in the vertically upward direction, $\bar{u}$ is the seepage velocity, ρ is the density, p is the pressure and ν is the kinematic viscosity of the saturated fluid.

The governing stability equation of the above mentioned fluid flow is given by

$$k^2\rho\left[U - c - \frac{i\nu\rho}{Kk}\right] = D\left[\rho\left(U - c - \frac{i\nu\phi^2}{Kk}\right)Dv - \rho v DU\right] - \frac{gD\rho\phi^2}{(U-c)}v. \qquad \ldots(4)$$

The non-dimensional form of equation (4) is given by

$$k^2\rho\left(U - c - iR_{D_0}^{-1}k^{-1}\right)v$$

$$= D\left[\rho\left(U - c - iR_{D_0}^{-1}k^{-1}\right)Dv - \rho v DU\right] + \frac{J_0\rho}{U-c}v, \qquad \ldots(5)$$

Where $R_{D_0}^{-1} = \frac{\nu\phi^2 d}{U_0 K}$ and $J_0 = \frac{g\phi^2\beta d}{U_0^2}$ in which $\beta = -\frac{D\rho}{\rho}$, d is the characteristic length and U_0 is the characteristic velocity. Here R_{D_0} is the ratio of the inertia force to the Darcy drag force called **Darcy-Reynolds** number in analogy with the ordinary Reynold number which is the ratio of the inertia force to the viscous force and is the ratio of the buoyancy force to the inertia force and is called the Richardson number.

The boundary conditions are

v=0 at y=0 and y=1. …(6)

Let

$$W_1 = U - c - iR_{D_0}^{-1}k^{-1} = W - iR_{D_0}^{-1}k^{-1}. \qquad \ldots(7)$$

Taking the transformation $v = W_1 F$ in (5), we get

$$k^2 \rho W_1^2 F = D\left[\rho W_1^2 DF\right] + \frac{J_0 \rho W_1 F}{W}. \qquad \ldots(8)$$

The corresponding boundary conditions are

$$F = 0 \text{ at } y = 0 \text{ and } y = 1. \qquad \ldots(9)$$

MATHEMATICAL ANALYSES:

For spatial stability $k(= k_r + ik_i)$ denotes the complex wave number and $\omega(> 0)$ the real frequency. The complex wave velocity c is given by

$$c = \frac{\omega}{k}. \qquad \ldots(10)$$

which gives

$$c_r = \frac{\omega k_r}{|k|^2} \text{ and } c_i = \frac{-\omega k_i}{|k|^2}. \qquad \ldots(11)$$

The phase velocity of the perturbations is defined as

$$c_p = \frac{\omega}{k_r}. \qquad \ldots(12)$$

The unstable modes will be characterized by $k_i \neq 0$

Multiplying equation (8) by $\overline{F}$ the complex conjugate of F, integrating over the flow domain and using the boundary conditions (9), we get

$$\int \rho W_1^2 \left(|DF|^2 + k^2|F|^2\right) - \int \frac{\rho J_0 W_1 |F|^2}{W} = 0. \qquad \ldots(13)$$

Separating the real and imaginary parts of (13) we get

$$\int \rho \left[\left(u - c_r - \frac{R_D^{-1}k_i}{|k|^2}\right)^2 - \left(c_i + \frac{R_D^{-1}k_r}{|k|^2}\right)^2\right] \times \left[|DF|^2 + \left(k_r^2 - k_i^2\right)|F|^2\right]$$

$$+ 4k_r k_i \left(c_i + \frac{R_D^{-1}k_r}{|k|^2}\right) \int \left(U - c_r - \frac{R_D^{-1}k_i}{|k|^2}\right)|F|^2$$

$$- \int \rho J_0 \left[(U - c_r)\left(U - c_r - \frac{R_D^{-1}k_i}{|k|^2}\right) + c_i\left(c_i + \frac{R_D^{-1}k_i}{|k|^2}\right)\right] \frac{|F|^2}{|W|^2} = 0, \qquad \ldots(14)$$

and

$$-2\int \rho \left(c_i + \frac{R_D^{-1}k_r}{|k|^2}\right)\left(U - c_r - \frac{R_D^{-1}k_i}{|k|^2}\right)\left[|DF|^2 + \left(k_r^2 - k_i^2\right)|F|^2\right]$$

$$+ 2k_r k_i \int \rho \left[\left(U - c_r - \frac{R_D^{-1}k_i}{|k|^2}\right)^2 - \left(c_i + \frac{R_D^{-1}k_r}{|k|^2}\right)|F|^2\right]. \qquad \ldots(15)$$

Equation (15) can be written as

$$\int \rho(c_i + Rc_r)(U - c_r + c_i R)\left[|DF|^2 + (k_r^2 - k_i^2)|F|^2\right]$$
$$-k_r k_i \int \rho\left[(U - c_r + c_i R)^2 - (c_i + Rc_r)^2\right]|F|^2$$
$$+\int \frac{\rho J_0}{2|U-c|^2}\left[c_i(U - c_r + c_i R) - (U - c_r)(c_i + Rc_r)\right]|F|^2 = 0,$$

where $R = \dfrac{R_D^{-1}}{\omega}$.

or $$\int p(U - c_r)|DF|^2 + \frac{R}{c_i}\int \rho\left[c_r(U - c_r) + c_1^2\right]|DF|^2 + R^2 c_r \int \rho |DF|^2$$
$$+\int \rho \frac{|k|^2}{c_p} U(U - c_p)|F|^2 + \frac{R_{D_0}^{-1} k_r}{c_i |k|^2}\int \rho(U - c_p)(|k|^2 - J_0)|F|^2 = 0. \quad \ldots(16)$$

From equation (16) it follows that $c_i < 0$ if

$$\left.\begin{aligned} U - c_p &> 0 \\ |k|^2 &> J_0 \end{aligned}\right\} \quad \ldots(17)$$

Thus we have the following result:

Theorem 1: Stable modes for $c_r > 0$ lie in the region given by

$$k_r > \frac{\omega}{b}$$

and $k_r^2 + k_i^2 > J_0$.

This result does not depend upon U' and $(\rho U')'$.

Taking substitution $F = W_1^{-1/2} G$ in (8) and dividing the resulting equation by $W_1^{1/2}$, we get

$$(\rho W_1 G')' - \rho W_1 k^2 G - \frac{(\rho U')'}{2} G - \frac{\rho U'^2}{4(W_1)} G + \frac{J_0}{W} G = 0. \quad \ldots(18)$$

Also taking the substitutions $F = W_1^{-1} H$ in (8) and dividing the resulting equation W_1, we get

$$(\rho H')' - (\rho k^2 H) - \frac{(\rho U')'}{W_1} H + \frac{\rho J_0}{W W_1} H = 0. \quad \ldots(19)$$

Multiplying equation (18) by $\overline{G}$, the complex conjugate of G and integrating over the flow domain, we get

$$\int \rho W_1\left[|G'|^2 + k^2 |G|^2\right] + \int \frac{(\rho U')'}{2}|G|^2 + \int\left(\frac{\rho U'^2}{4W_1} - \frac{\rho J_0}{W}\right)|G|^2 = 0. \quad \ldots(20)$$

Similarly multiplying equation (20) by $\overline{H}$, the complex conjugate of H and integrating over the flow domain, we get

$$\int \rho W_1\left[|H'|^2 + k^2 |H|^2\right] + \int \frac{(\rho U')'}{2}|H|^2 + \int\left(\frac{\rho J_0 |H|^2}{W W_1}\right) = 0. \quad \ldots(21)$$

The imaginary part of (20) is given by

$$\int \rho |G'|^2 + \frac{Rc_r}{c_i}\int |G'|^2 + \frac{k_r^2}{c_r}\int \rho(2U - c_p)|G|^2 + \frac{Rc_r}{c_i}\int\left(|k|^2 - \frac{U'^2}{4|W_1|^2}|G|^2\right)$$

$$+\int\left(\frac{J_0}{|W|^2} - \frac{U'^2}{4|W_1|^2}\right)\rho|G|^2 = 0. \qquad \ldots(22)$$

Now if the conditions

$$2U - c_p > 0,$$

$$|k|^2 - \frac{U'^2}{4|W_1|^2} > 0,$$

and $$\frac{J_0}{|W|^2} - \frac{U'^2}{4|W_1|^2} > 0,$$

hold everywhere in the flow domain, then necessarily c_i should be negative which implies

$$k_r > \frac{\omega}{2b'}$$

$$\left(k_r - \frac{\omega}{U}\right)^2 + \left(k_i - \frac{\omega}{U}\right)^2 > \frac{U'^2}{4b^{2'}}$$

and $$\left(k_r - \frac{\omega}{U}\right)^2 + \left(k_i - \frac{\omega R}{U}\frac{4J}{1-4J}\right)^2 < \frac{\omega^2 R^2}{U^2}\cdot\frac{4J}{(1-4J)^2}$$

where $J = \frac{U'^2}{J_0} < \frac{1}{4}$.

Thus we have the following result:

Theorem 2: For $k_r > 0$ the stable modes lie in the region given by

$$k_r > \frac{\omega}{2b}$$

$$\left(k_r - \frac{\omega}{U}\right)^2 + \left(k_i - \frac{\omega}{U}\right)^2 > \frac{U'^2}{4b^2}$$

And $$\left(k_r - \frac{\omega}{U}\right)^2 + \left(k_i - \frac{\omega R}{U}\cdot\frac{4J}{1-4J}\right)^2 < \frac{\omega^2 R^2}{U^2}\cdot\frac{4J}{(1-4J)^2}.$$

This theorem confirms the fact that the porous media with high Darcy resistance has a stabilizing effect The imaginary part of (21) is given by

$$-2c_i\left[\frac{k_r^2}{c_r}\int \rho|H|^2 - \int\frac{(\rho U')'|H|^2}{2|W_1|^2} + \int\frac{\rho J_0(U - c_r)}{|W|^2|W_1|^2}|H|^2\right]$$

$$+R\left[\int\frac{(\rho U')'}{|W_1|^2} - \int\left[\frac{\rho J_0\{(U - c_r)c_r + c_i^2\}}{|W|^2|W_1|^2}\right]|H|^2\right] = 0. \qquad \ldots(23)$$

If ($\rho U'$) is negative and $\left(U - c_r \right) > 0$ everywhere in the flow domain, then equation (23) holds good if c_i is negative which implies that k_i is positive.

But $U - c_r > 0$ everywhere in the flow domain

$$\Rightarrow c_r < U \text{ for all } y$$

$$\Rightarrow c_r < U_{\max} = b$$

$$\Rightarrow \frac{\omega k_r}{|k|^2} < b$$

$$\Rightarrow |k|^2 > \frac{\omega k_r}{b}$$

$$\Rightarrow \left(k_r - \frac{\omega}{2b} \right)^2 + k_i > \left(\frac{\omega}{2b} \right)^2.$$

Thus we have the following theorem.

Theorem 3: If $(\rho U')'$ is negative throughout the flow domain, then for spatially decaying stable modes with $k_r > 0$, (k_r, k_i) must lie outside the circle whose centre is $\left(\frac{\omega}{2b}, 0 \right)$ and radius $\frac{\omega}{2b}$.

CONCLUDING REMARKS:

Here the spatial stability of density stratified shear flows through a porous medium has been discussed. The spectrum of Eigen values has been obtained. Under certain conditions applied on density and velocity profiles, theorems have been established.

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